Geometric Modeling and Violin Design
Jordan Hess

Introduction

This paper presents a design framework that can accurately reproduce early Italian molds and instruments, including how to position bouts and corners, while at the same time accounting for the asymmetries found in period molds and violins. Shortly after I began making violins in 2010, I became interested in the uncovering the geometric models that 16th century violinmakers used. I started playing with mathematical concepts and patterns, trying to make sense of how early makers designed their instruments. This paper outlines one possible method for designing violins, which I call the internal frame approach. Using basic geometry that was widely known during the Amati dynasty, this method can be used to accurately replicate the violins from Cremona and surrounding regions during this era. It can also be used for developing new violin designs.

The ideas presented in this paper build on the work of Francois Denis[1] and Kevin Kelly[2], each of whom furthered my thinking about this type of modeling. Denis's excellent writing on historical architecture and geometry helped me better understand the cultural context of Cremona and the role that geometry played in instrument design. The importance of ratios and geometry in Renaissance art, architecture and design is well established.

Kelly introduced me to the use of vesicas in violin design, which provided the building blocks for the approach outlined in this article. My method differs from Kelly's in the way in which the size of the vesicas is calculated and in how the vesicas are positioned in relationship to one another. In Kelly's method, the shape of the instrument is determined by positioning the center bouts in a tangential relationship with the upper and lower bouts. In my method, the upper, lower and center bouts are positioned on an internal framework and the relationship between the center bouts and the upper and lower bouts is rarely tangential.

One of the biggest challenges in creating a geometrical template for instrument design is to account for the asymmetries that are pervasive in the molds and instruments of that era. Figuring out a methodology that accounts for commonly

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[1] Denis, Francois. Traite De Lutherie – The Violin and the Art of Measurement.
[2] Kelly, Kevin. https://www.youtube.com/watch?v=G9H7oJVj2Ns.

seen distortion between the upper and lower bouts, while also replicating the precise shapes, positioning and angles elsewhere in the instrument drove my interest in the subject, as did my interest in creating a method for new instrument design.

The design framework is discussed in four parts: The first section introduces vesicas and shows how they are used in designing the upper and lower bouts. The second part discusses the framework for setting the bouts. The third section outlines the process of creating the C bouts and corners. This article ends with examples of the model superimposed upon photos of several violins.

Vesicas

Like Kelly, I hypothesize that vesicas could have been the building blocks of early violin design. A vesica is a geometrical shape that is the intersection of two circles of the same radius positioned on the axis of a larger circle such that the outer edges of the inner circles are tangential to the larger circle on both sides. In this article I refer to vesica ratios by the space each of the three sections (1-1-1 in Figure 1) occupy on the axis of the larger circle.

In the case of the vesica piscis, the two inner circles intersect each other in such a way that the center of each circle lies on the perimeter of the other circle. This divides the center axis of the larger circle into 3 equal parts, giving it a 1-1-1 ratio. By positioning the inner circles closer or farther apart, vesicas with different ratios can be made. Figure 2 shows a vesica with a 1-2-1 ratio.

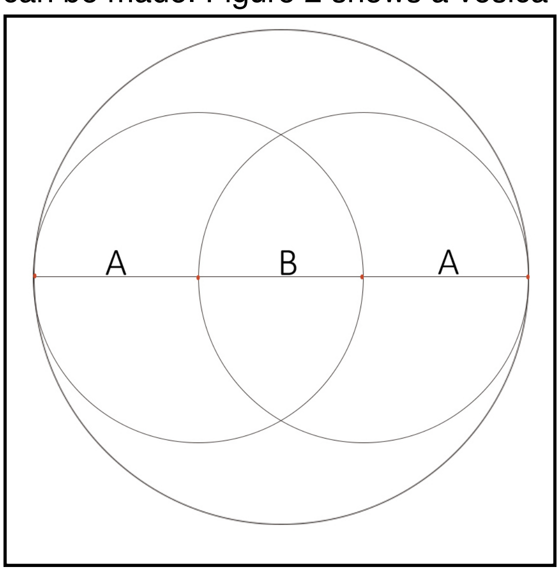

Figure 1. Vesica Piscis.
The A-B-A ratio equals 1-1-1.

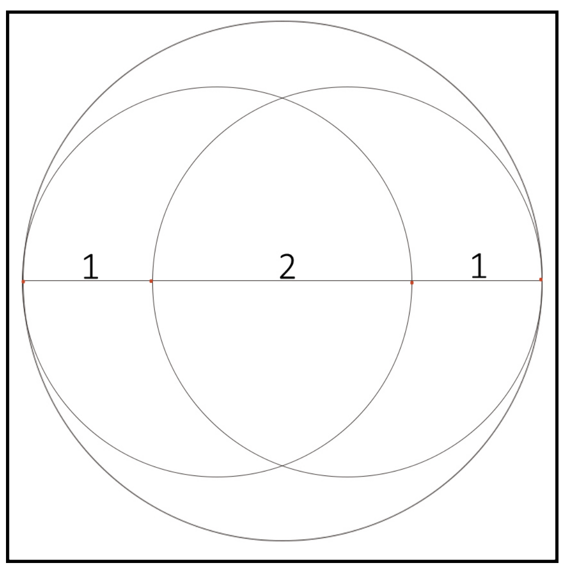

Figure 2. A vesica with a 1-2-1 ratio.

Luthiers can use vesicas to make both the upper bout and lower bout. The lower bout is almost always the vesica piscis using a 1-1-1 ratio, but the upper bout can vary, typically employing a vesica with a 1-2-1 or 2-5-2 ratio.

When drawing a vesica with a specific ratio, first decide the total width of the vesica, and draw a line of the same length where you are going to position the vesica. This center line is the diameter of the larger circle. Next, add the total number of segments. The number of segments is equal to the sum of the ratio. For example, in the case of a 1-2-1 vesica, the number of segments is four (1 + 2 + 1 = 4). Divide the center line by the number of segments. For example, if the center line is 80 mm in length the each segment will be 20mm. Mark the segments along the line (Figure 3).

Determine the number of segments for each of the inner circles. In a 1-2-1 vesica the total number of segments for each inner circle is three (1+2). The diameter of the two inner circles is measured from the edge of the vesica (tangential to the inside of the larger circle) to the point on the axis equal to the number of segments of that circle (Figure 4). Do the same for the other inner circle but position it on the other side of the larger circle.

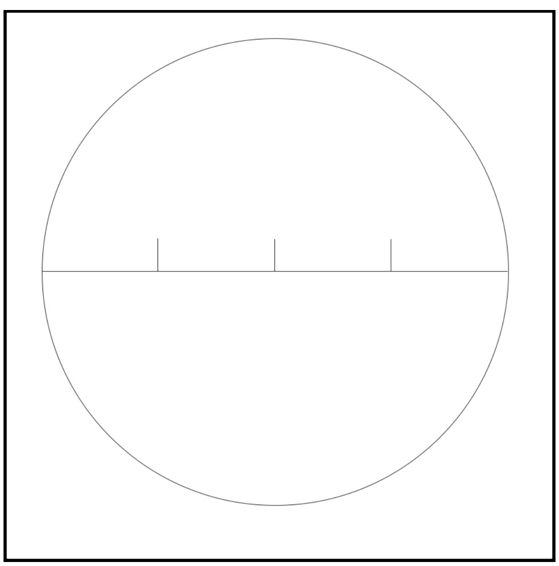

Figure 3. Larger circle of the vesica with center line divided by the sum of the ratio.

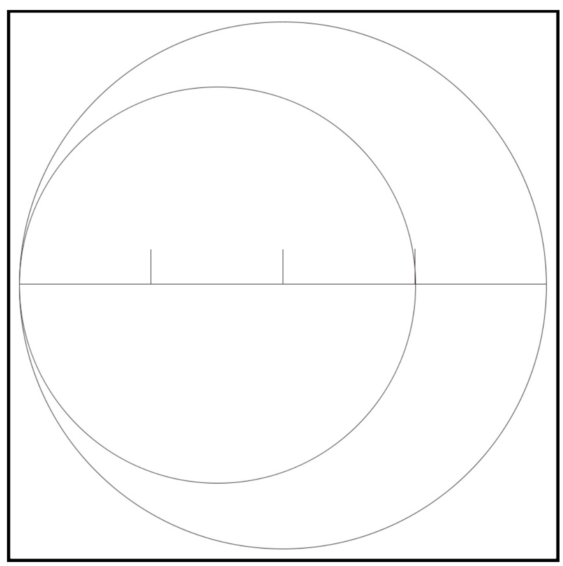

Figure 4. The first of the inner circles is drawn so that it is tangential to the outer circle and goes through its axis point.

Vesica Modifications

Most vesicas used in classic instrument design were unmodified, but in some instruments they were modified. The modifications impact the outside shape of the corners and the overall aesthetic of the instrument. The modifications always take place on the edge of the vesica that lies closest to the center of the instrument. There are two types of modification: internal and external. Internal modifications are almost exclusively used in the lower bout. Outside modifications are usually used in the upper bout, but they can be used in the lower bout as well (as in the case of the long pattern Strads). The modifications take place on the half of the vesica closest to the center of the instrument.

In an internal modification, a vesica is drawn. In this example using a 1-1-1 ratio (Figure 5). Next, the halfway points are found between the center of the vesica (Point A) and the center of each of the inner circles (Points B). The midway points between A and B (Points C) become the axis for modifying the top and bottom of the vesica.

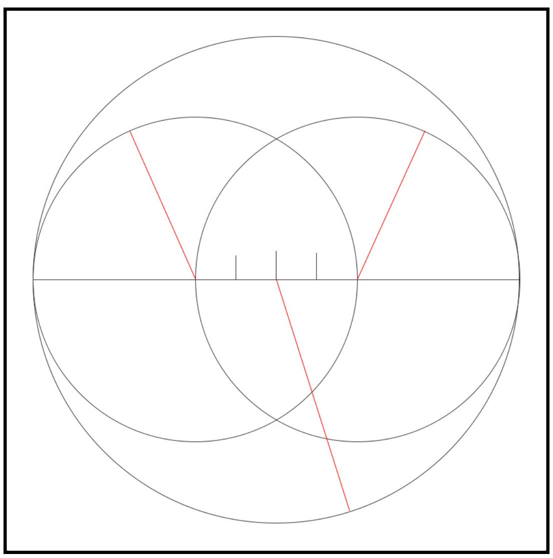

Figure 5. Points C become the axis for modifying the vesica.

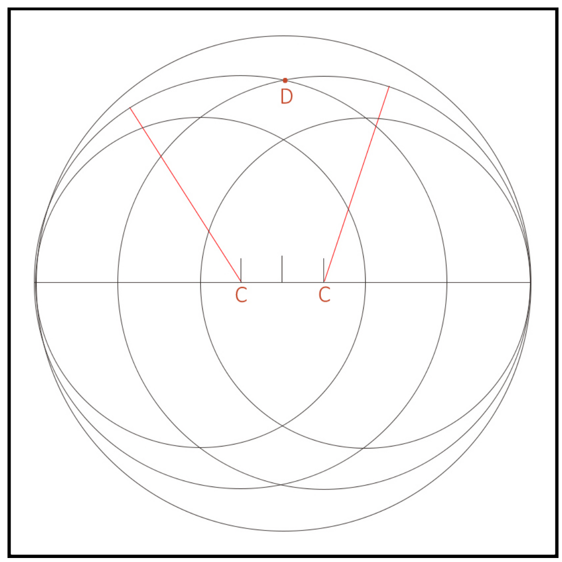

Figure 6. Internal modification of a vesica.

Two arcs are then drawn, with their centers positioned at each of these midway points (Figure 6), so that they are tangential to the nearest edge of the vesica. In other words, the radius of each new arc extends from the new center point (Points C) to the closest edge of the larger circle. The resulting arcs intersect near the inside of the top and bottom of the outer circle. These new arcs become the new top and new bottom of the vesica, creating an oblong rather than circular structure.

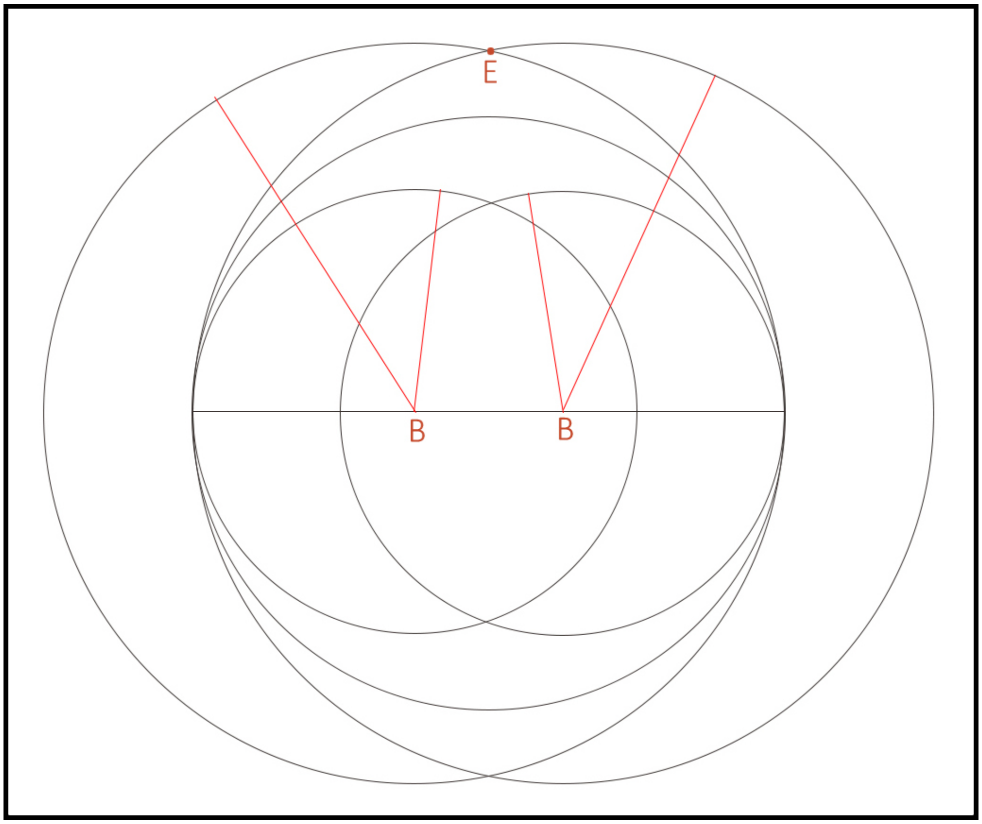

Figure 7. External modification of a vesica.

In an outside modification, a vesica is drawn using a 1-2-1 ratio (Figure 7). This time, the radius for the new arc uses the same midpoint as that used to draw the inner circles (Points B) and extends along the axis line to the farthest edge of the outer circle. Each arc is drawn so that it is tangential to the outside circle on its opposite side. The arcs are extended to the point where they cross one another at the top and bottom of the vesica (Point E). Where the internal modification reduces the top and bottom of the vesica, the external modification extends them.

Internal framework

The vesicas are positioned on an internal framework, to create the skeleton of the instrument. This internal framework consists of a vertical line bisected by three horizontal lines (Figure 8). The vertical line becomes the center of the instrument. The top line is the widest point of the upper bout (upper bout line or UB), the middle line is the narrowest point of the C bout (center bout line or CB), and the bottom line is the widest point of the lower bout (lower bout line or LB).

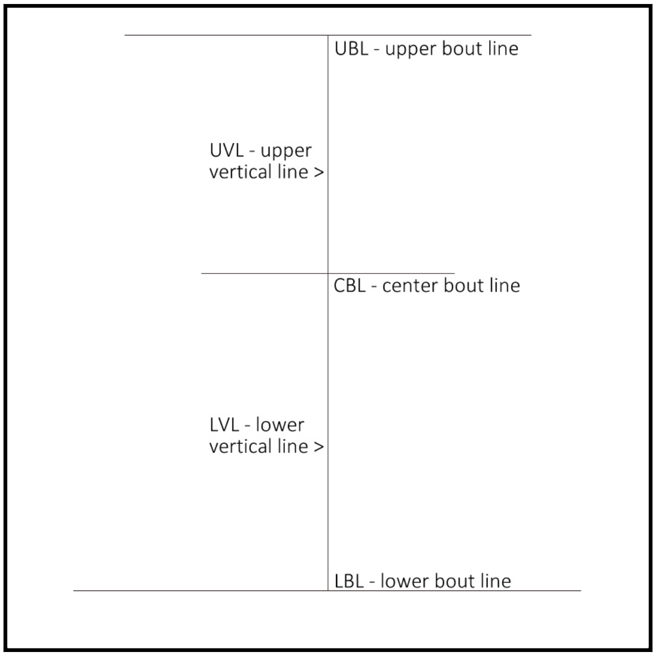

Figure 8. Internal Framework Structure.

Four critical ratios are used in creating the framework. These ratios determine the relative length and width of the instrument.

LVL:UVL Height Ratio
The distance between the lower bout line and center bout line compared with the distance between the center bout line and upper bout line (LVL: UVL). A 4:3 ratio is by far the most common ratio found in early Italian instruments, though other ratios, such as 7:5 are also used.

LVL: ½ LBL Width Ratio
The distance between the lower and center bout lines compared to half of the width of the lower bout line. (LVL: ½ LBL) A 5:4 ratio is typically used, though in most Strads a 6:5 ratio is used.

UBL:LBL Width Ratio
The width of the upper bout line compared with the width of the lower bout line (UBL: LBL). A 4:5 ratio is standard.

CBL Width Ratio
The width of the center bout line is typically one half the width of the lower bout line or two-thirds the width of the upper bout line (CBL: ½ LBL or CBL: 2/3 UBL). In some cases the difference between these ratios is split.

FRAME RATIOS						
Maker	Model	City	LVL:UVL	LVL:1/2LBL	UBL:LBL	CBL Width
Strad	PG	Cremona	4:3	6:5	4:5	1/2 LBL
Strad	P	Cremona	4:3	6:5	4:5	1/2 LBL
Strad	G	Cremona	7:5	5:4	4:5	1/2 LBL – 2/3
N. Amati	Grand Pattern	Cremona	4:3	5:4	4:5	1/2LBL
A. Amati	Violin	Cremona	4:3	5:4	4:5	1/2 LBL

Table 1 shows examples of different ratios used by different makers and in different models.

The following exercise uses the ratios of the Amati Grand Pattern.

The internal framework begins with a vertical line. For a violin, the line is least 220 millimeters long, although the whole length will not be used. A second line is drawn that is perpendicular to the vertical line and crosses at the bottom end, creating an upside down “T” (Figure 9). This becomes the lower bout line. The length of this lower bout line is equal to the width of the lower bout of the mold or template and should be centered on the vertical line (in this example use a 200mm line).

To determine the placement of the center bout line, refer to the LVL: ½ LBL ratio. In our example, the Amati Grand uses a 5:4 ratio. Next the width of the lower bout line is divided from the point where the two lines meet to its edge into four equal parts (in our example these are each 25mm). The length of one of the four segments is used to create five segments on the vertical line. Through this top point, a line is drawn that is perpendicular to the vertical line (Figure 10). This is the center bout line. The CBL Width is a 1:2 ratio. The length of this line is half the width of the lower bout line or 100mm in length (100mm: 200mm = 1:2).

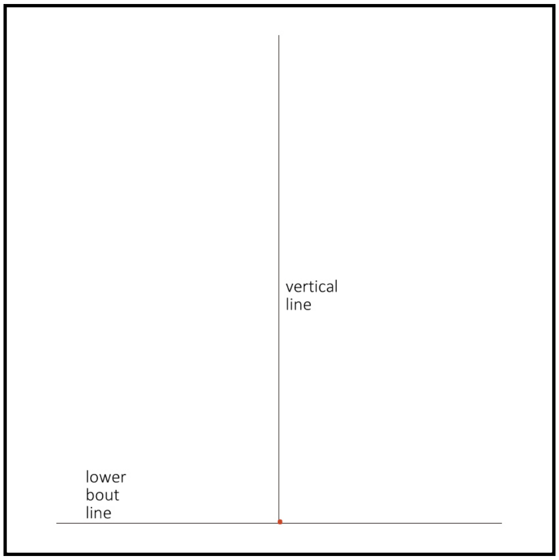

Figure 9. Placing Lower Bout Line.

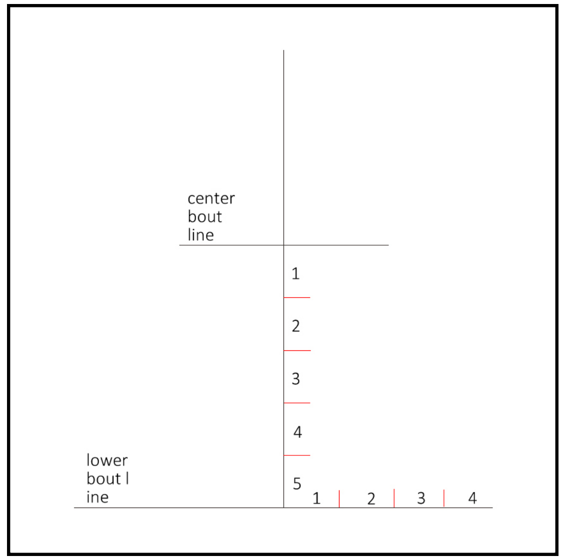

Figure 10. Placement of Center Bout. Line

To determine the placement of the upper bout line, again refer to the LBL:UBL ratio. The Amati Grand Pattern uses a 4:3 ratio. The length of the vertical line that is between the lower and center bout lines is divided into 4 equal segments. Each segment becomes a unit of measure. The upper bout line is located three units above the center bout line.

The UBL:LBL ratio is used to determine the width of the upper bout line. Since our example uses a 4:5 ratio, the upper bout line will be 160mm in width (160mm: 200mm = 4:5). As with the other two horizontal lines, this one should be centered on the vertical line. The vertical line can now be trimmed off at this point (Figure 11).

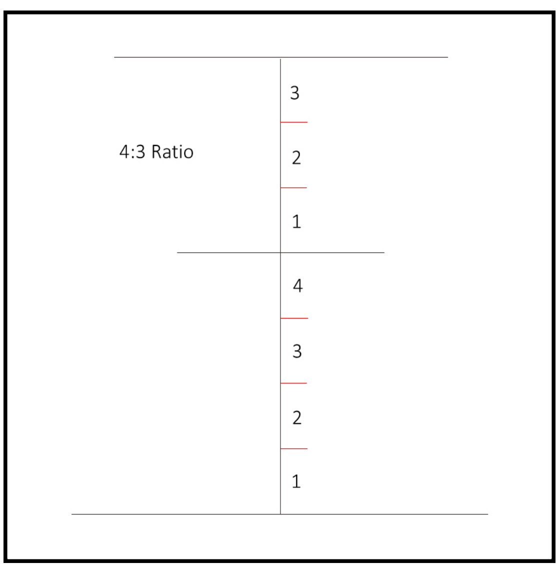

Figure 11. Placement of Upper Bout Line.

Putting it all together

Once the framework is drawn, vesicas are located on the upper and lower bouts, and the center bout arcs are drawn. As mentioned at the beginning of article when discussing vesicas, a variety of ratios are used for sizing vesicas. Table 2 shows vesica ratios for several classic instruments.

Continuing with the Amati Grand example, a vesica is drawn on the lower bout line (in this case using a 1-1-1 ratio) that is the same width as the lower bout line. The axis of the vesica is positioned on the lower bout line (Figure 12). In the example, the lower bout line is 200mm, so the radius of each of the internal circles is 66 2/3 mm. With 1-1-1 ratios, each circle is 2/3 of the width of the vesica.

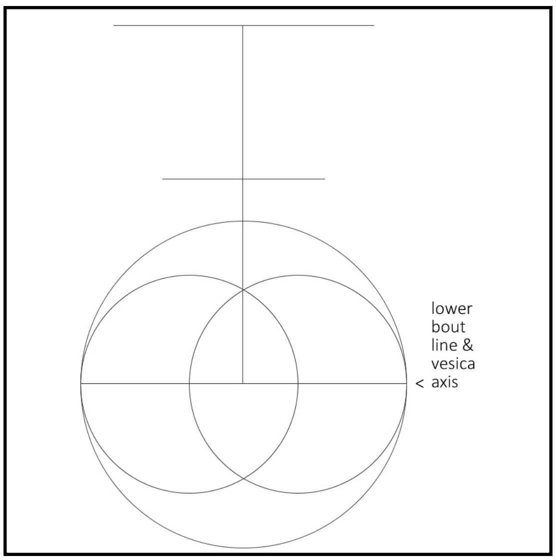

Figure 12. Placing Lower Vesica.

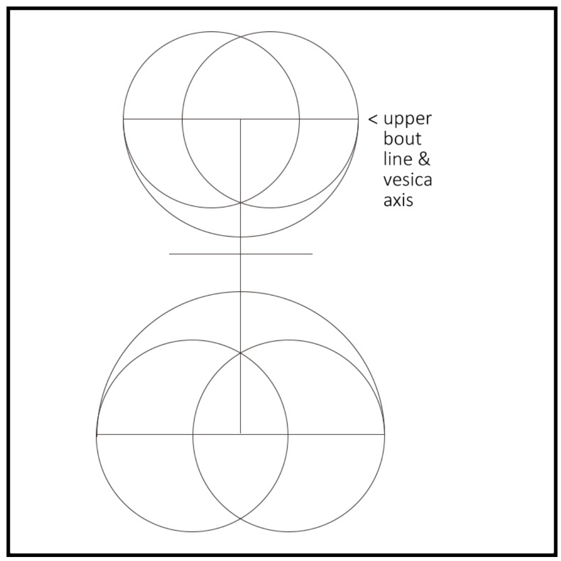

Figure 13. The upper bout vesica is added, and the extra is erased from both the upper and lower bout.

For the upper bout a vesica using a 1-2-1 ratio is drawn, with its axis on the upper bout line. In the example, the upper bout line is 160mm, so the radius of each circle is 60mm. Each circle in a 1-2-1 ratio is 3/4 of the width of the vesica. (Figure 13).

VESICA RATIOS					
Maker	Model	City	Lower Vesica	Upper Vesica	C Bout Arc
Strad	PG	Cremona	1-1-1	2-5-2	lower bout radius
Strad	P	Cremona	1-1-1 w/ internal mod	2-5-2	
Strad	G	Cremona	1-1-1	2-5-2	
N. Amati	Grand Pattern	Cremona	1-1-1	1-2-1 with external mod	upper bout radius
A. Amati	Violin	Cremona	1-1-1	1-3-1	upper bout radius

Table 2 shows examples of different ratios used in the creation of vesicas.

<u>Explaining Asymmetries</u>

The use of an internal framework upon which vesicas are drawn could account for the asymmetries found in early Italian instruments. Accurately drawn vesicas that are located on a frame that is slightly out of square would create asymmetrical results. Whether this is the result of wooden measuring tools, such as squares, that are ever-so-slightly off-square, or a lack of precision in placing the vesicas on this framework is difficult to determine. What is clear from looking at early Italian molds and instruments is that the vesicas are drawn with a precision that is lacking in the framework. The result is that the relationship between the upper and lower bouts of many period instruments is skewed, resulting in asymmetrical instruments. An internal frame approach where the upper and lower bout lines are not precisely parallel with one another provides a plausible explanation for instrument asymmetry.

The examples (Photo 1 and Photo 2) show the upper and lower bouts on the Strad PG mold. A drawing of the appropriate vesica is superimposed on each photo and aligned with the vertical center line of the mold. In each instance the bout is skewed, with one corner lower than the other.

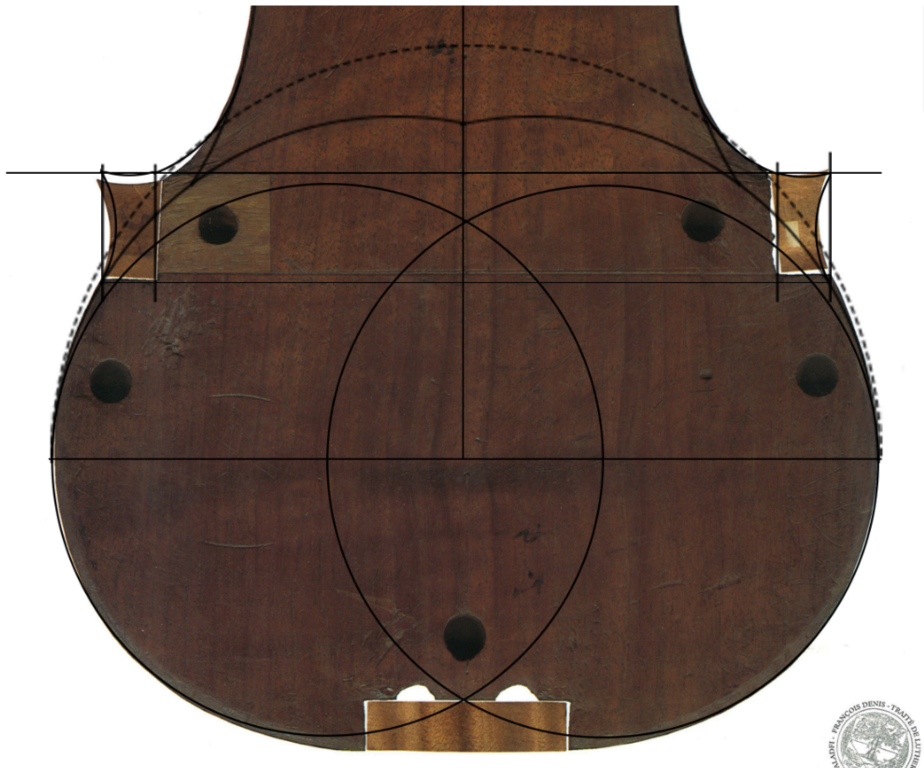

Photo 1 shows the lower bout of the Strad PG. Note how the corners are asymmetrical.

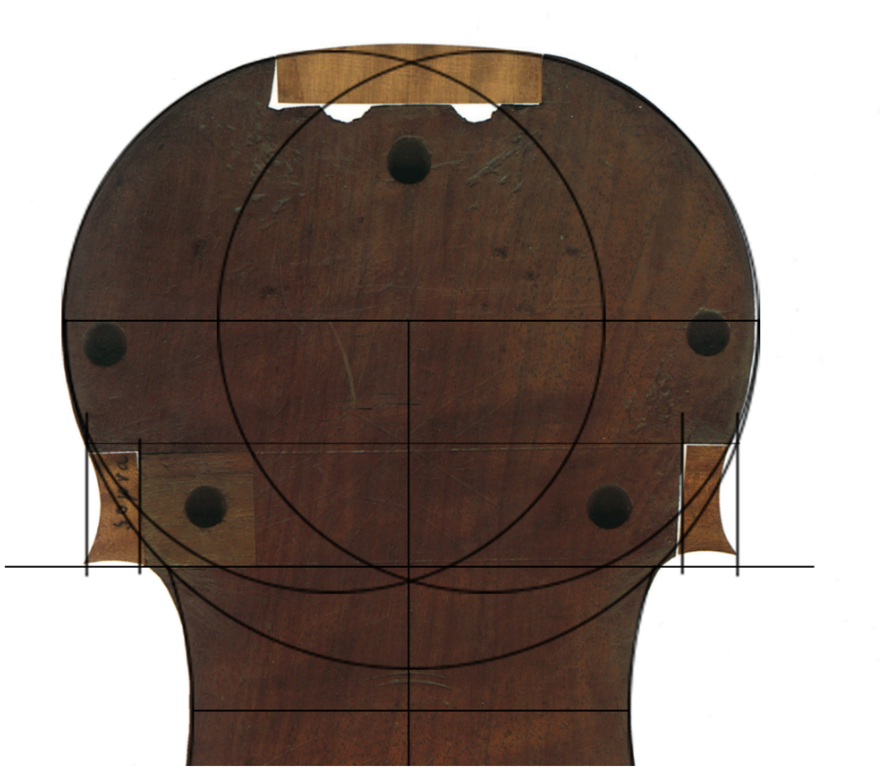

Photo 2 shows the upper bout of the Strad PG, with asymmetrical corners.

Modifying Vesicas

Any modifications to the vesicas are now made. In this example, an external modification on the upper bout is made.

The ends of the upper and lower bouts are drawn. (Figure 14). For the upper end, the point on the vertical line where the outside arc of the lower vesica intersects it is located (Point A). An arc is drawn that is tangential to the inside circles of the upper vesica (Line A). This process is repeated with the lower end. This time with the center point where the outside arc of the upper vesica crosses the center line (Point B), and is tangential to the inside circles of the lower bout (Line B).

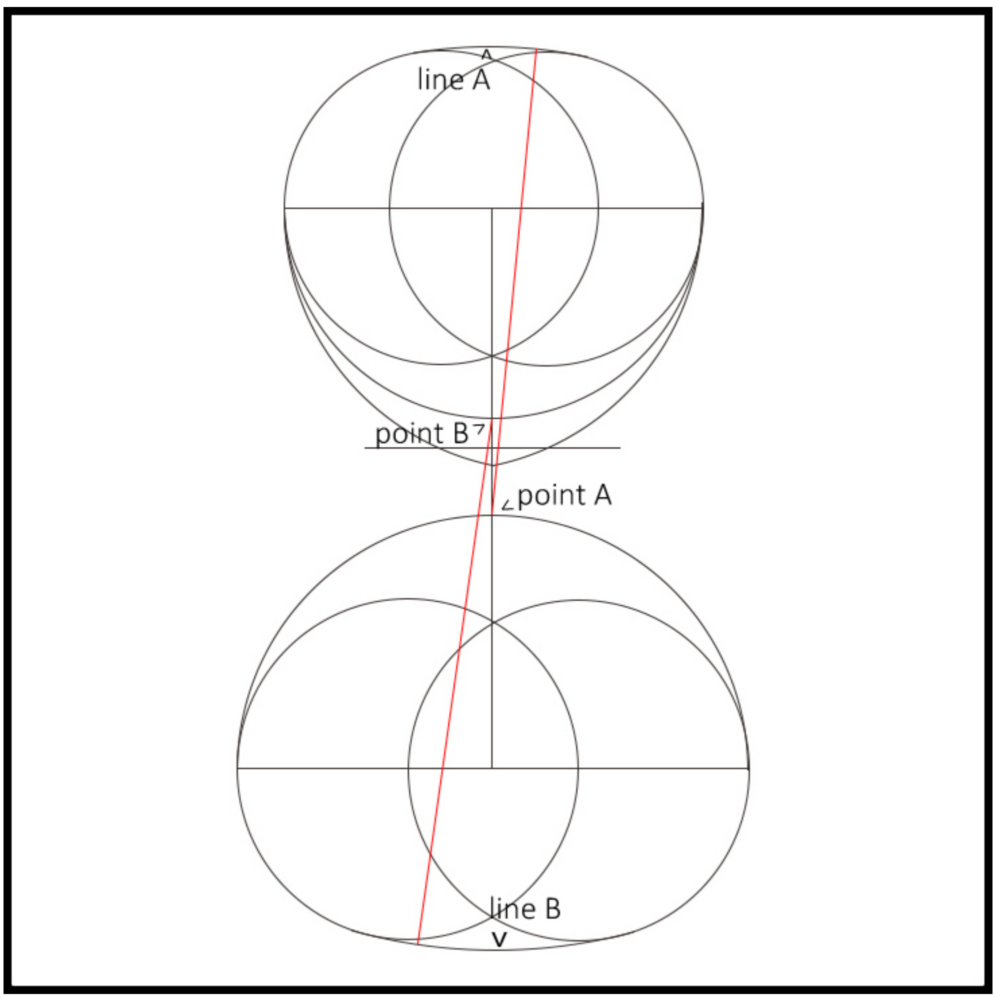

Figure 14. The caps on the upper and lower bouts.

Next, the focus shifts to the center bout. The center bout line is centered on the vertical line. The Amati Grand Pattern uses a 1:2 ratio of the width of the center bout line to the width of the lower bout line, resulting in a 100mm line. An arc is drawn on each side of the instrument that touches the end point of the center bout line and has the same radius as the upper bout. If the center bout line were extended, it would evenly divide the imaginary circle that is created by this arc. (Figure 15). The arc can also be drawn using the radius of the lower bout.

These arcs connect the upper and lower vesicas, creating a shape that makes up the air volume of the instrument. The addition of corners does not impact the air volume of the instrument but it plays a critical role in giving the instrument its final shape and look.

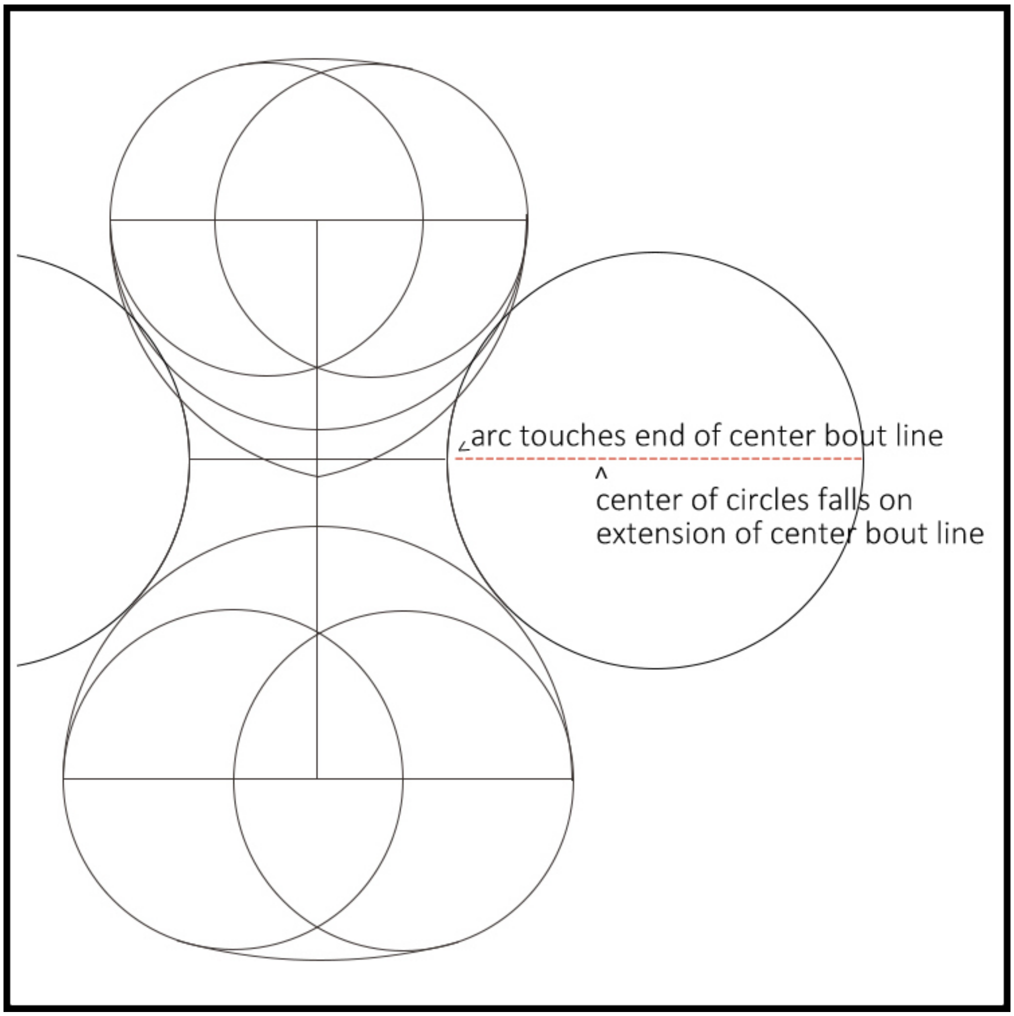

Figure 15. Finished upper and lower bout vesicas with the center bout arcs.

The Corners

As with the frame, ratios play a critical role in locating and designing corners. Three critical ratios are used:

Upper Vertical Constraint
This line is located between the center and upper bout lines. The distance between this line and the center and upper bout lines is typically a 2:3 or 3:5 ratio.

Lower Vertical Constraint
This line is located between the center and lower bout lines. The distance between this line and the center and lower bout lines is typically a 4:5 or 5:8 ratio.

CORNER RATIOS				
Maker	**Model**	**City**	**Upper Vertical Constraint**	**Lower Vertical Constraint**
A. Amati	Violin	Cremona	2:3	4:5
N. Amati	Grand Pattern	Cremona	2:3	4:5
Strad	PG	Cremona	3:5	3:4
Strad	P	Cremona	3:5	3:4
Strad	G	Cremona	3:5	3:4

Chart 3 shows examples of different ratios used in locating the corners on instruments.

Locating the corners is begun by placing horizontal lines that will be the first constraint of the corners.

The upper vertical constraint line is located using a 2:3 ratio. The distance between the center bout line and the upper bout line is divided into five equal parts. The upper vertical constraint line is drawn at the top of the 2nd segment above the center bout line and three segments below the upper bout line (Figure 16).

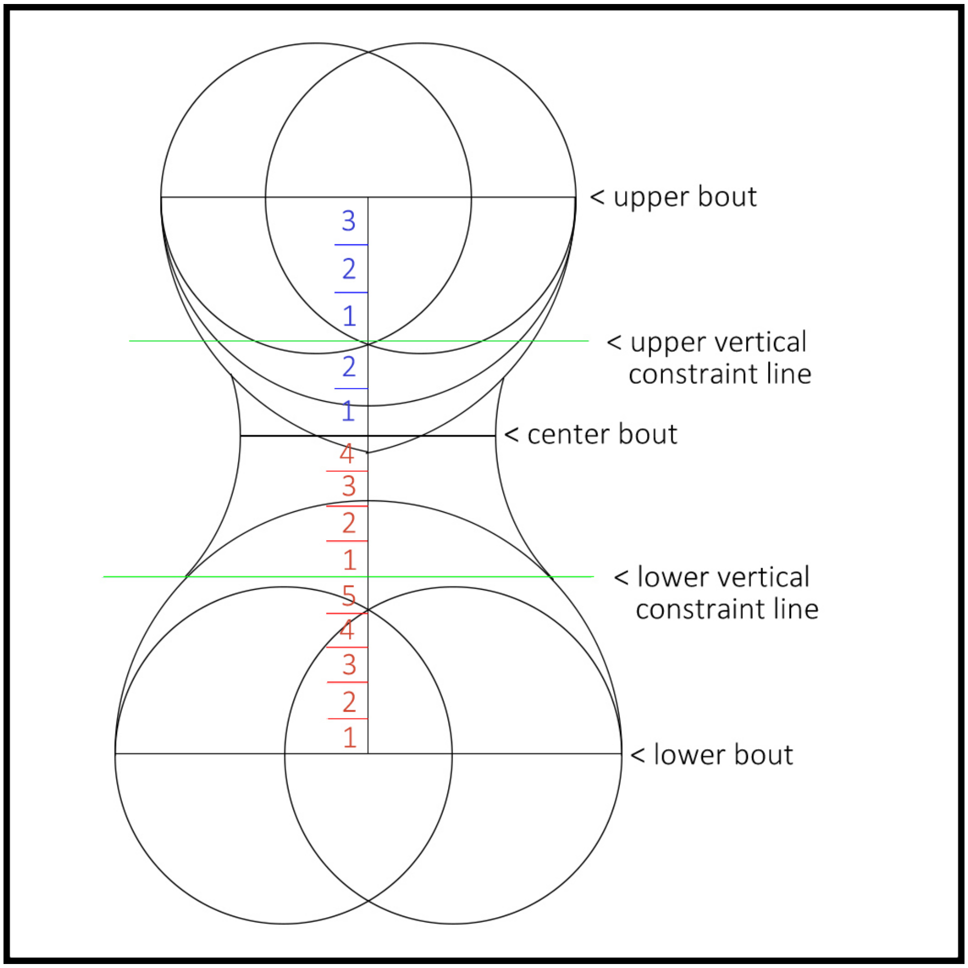

Figure 16. The vertical parameters for the corners are determined.

With the lower vertical constraint line a 4:5 ratio is used. The distance between the center bout line and the lower bout line is divided into nine equal parts. The lower vertical constraint line is drawn so that it intersects the point that is four units below that center line and five units about the lower bout line.

Next, the difference in width between the widest part of the lower bout and the narrowest part of the center bout is divided into four equal parts. Vertical lines are drawn through these points of division. The outermost line (Line A) will go through the lower corner constraint, the middle line (Line B) will go through both the lower and upper and the inner line (Line C) will go through the upper (see Figure 17).

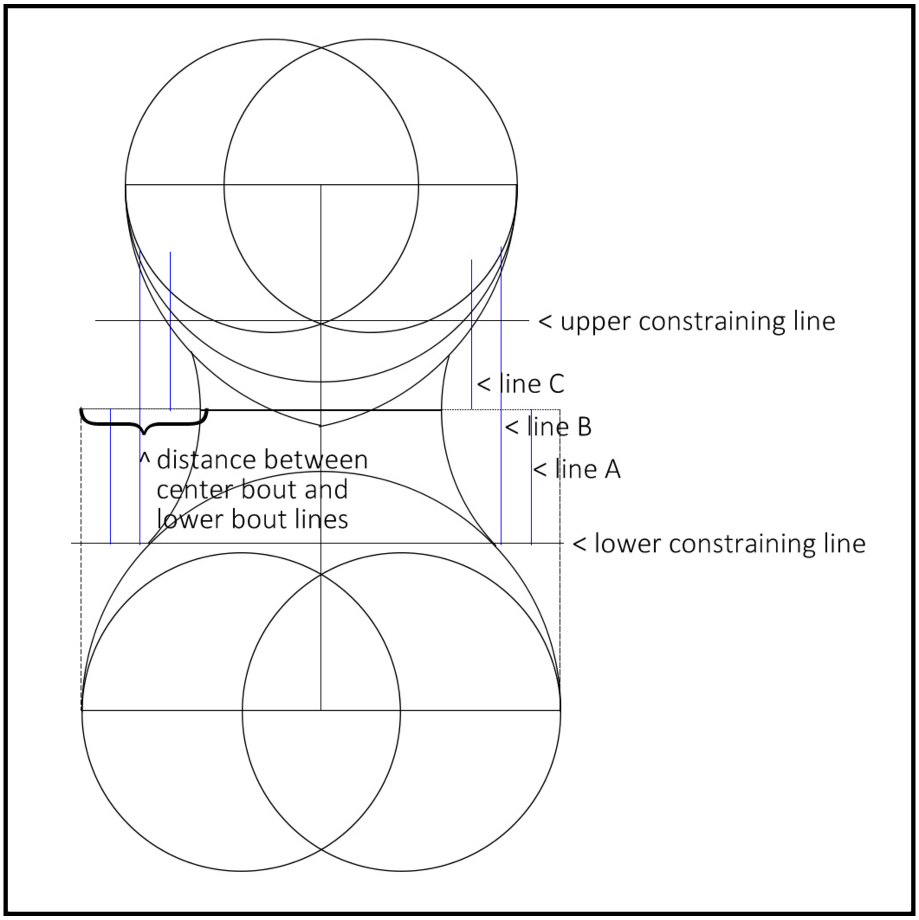

Figure 17. The horizontal parameters for the corners are determined.

The next step is to draw the arcs for each of the corners. In this article, I describe what I refer to as the standard method, which fits the instruments made by the Amatis and followers of the Amati school. A second method, which I call the modified method, fits instruments made by Stradivari. I plan to outline the modified method in another article.

Standard Method for Drawing Corner Arcs

In the standard method, the radius of the lower vesica is divided into four parts. The radius of the first arc is equal to one of these parts. The outside arcs of all four corners as well as the inside arc of the lower corners use this size arc. The outside arcs of the corners are drawn so that the arc passes through the outside constraining point for that particular corner and so it is tangential to the arc of the vesica used for the outline in that particular section (see Figure 18). The lower outside arc touches the lower constraining line at the point where Line A crosses it. The upper outside corner arc touches the upper constraining line at the point where Line B crosses it.

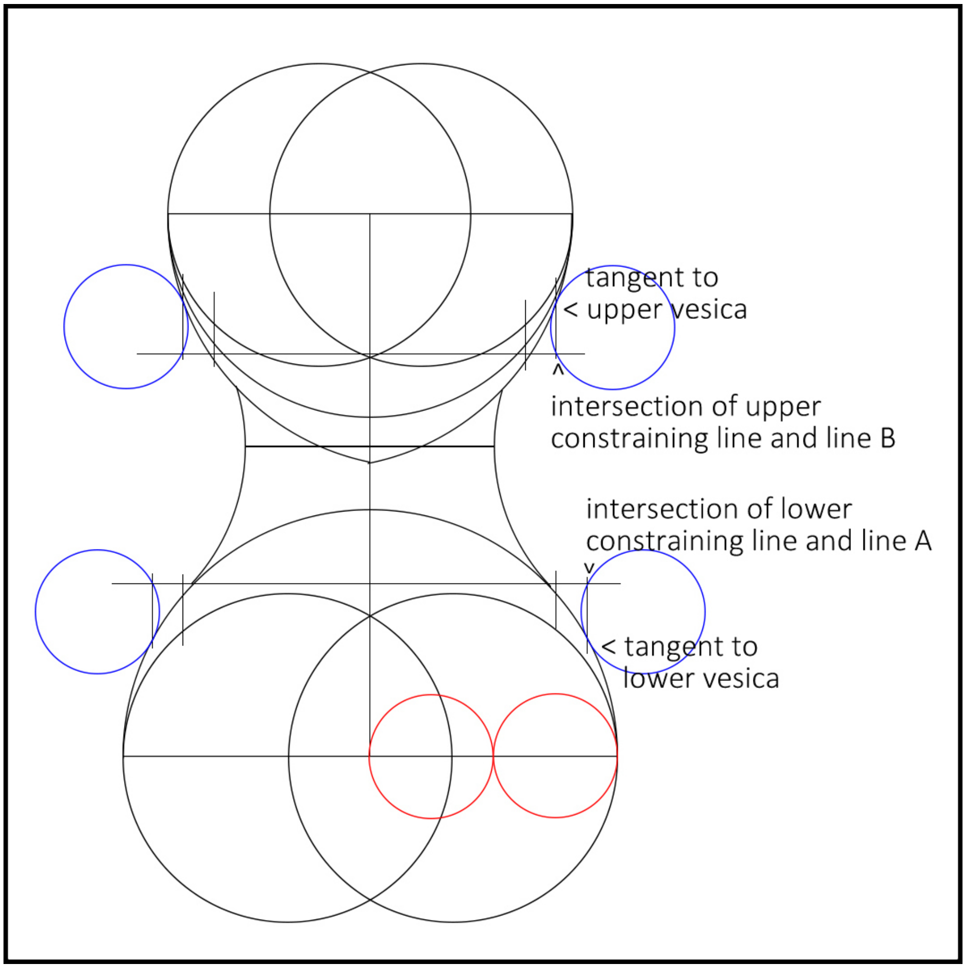

Figure 18. The outer arcs for the corners are drawn.

The inside arc for the lower corners is of the same radius as the arc for the outside of the corners and passes through the same point as the outside arcs (intersection of Line A and lower constraining line), but it is tangential to the center bout arc instead of the lower bout (see Figure 19).

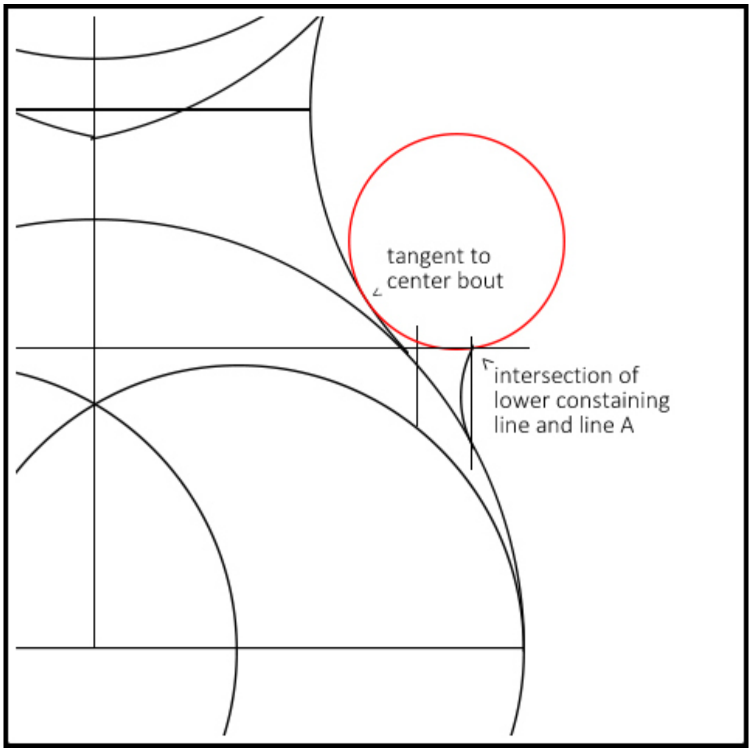

Figure 19. Positioning the inside arc of the lower corners.

The radius of the inside arc for the upper corners is found the same way as the other radius, but using the upper bout this time. It is drawn the same way that the inside lower arc is drawn (Figure 20). With the completion of these arcs, the external shape of the violin is competed.

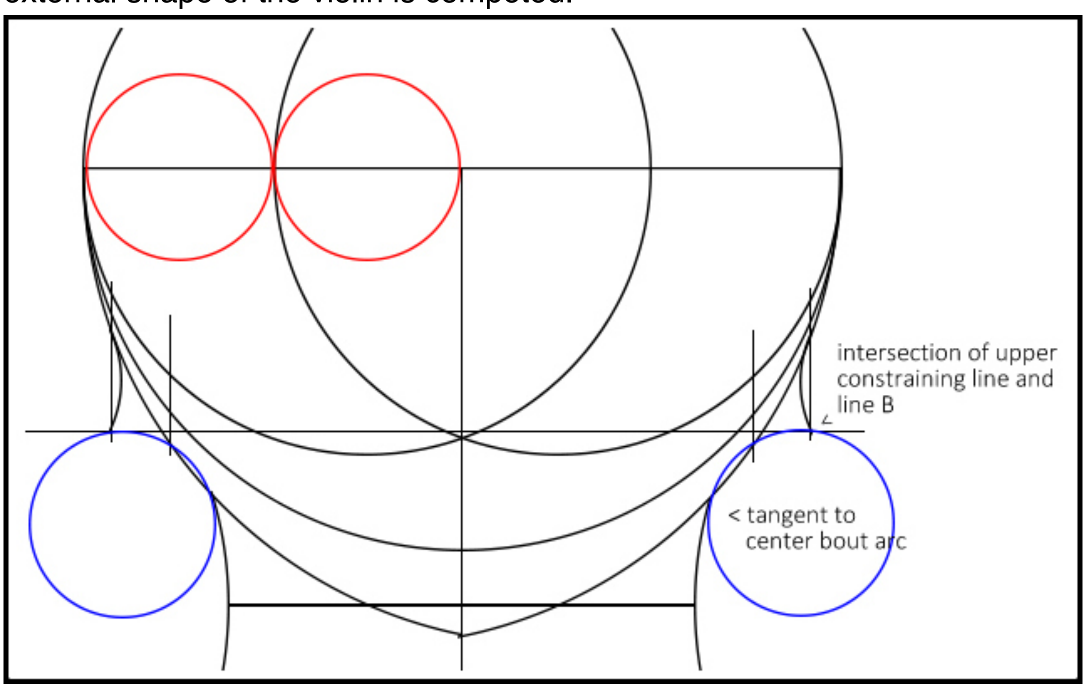

Figure 20. Positioning the inside arc of the upper corners.

Examples – Applying the Model to Actual Instruments

The example in this article creates the outline for an Amati Grand Pattern instrument. With adjustments to the ratios, any of the instruments from Cremona or from surrounding regions can be replicated. In addition to violins, the method works for cellos, violas, lutes and other period stringed instruments. Since the Amati Grant Pattern mold no longer exists, I am applying the method to another mold. The following illustration show the model superimposed upon the Strad G mold (Figure 21). The outline created by this method fits almost exactly on this mold, demonstrating how such an approach could be used to design the Strad G mold.

While there remains much to be learned about the working methods of early luthiers, I believe this model provides a viable explanation of how instruments could have been designed using tools and mathematics that had currency during that period. It succeeds in creating a design template that can accurately reproduce early Italian molds and instruments in a manner that accounts for the asymmetries that are found. The method accurately places corners and bouts in relationship to one another. Finally, this tool can be used not only to replicate the designs of early instruments, but also to design new instruments.

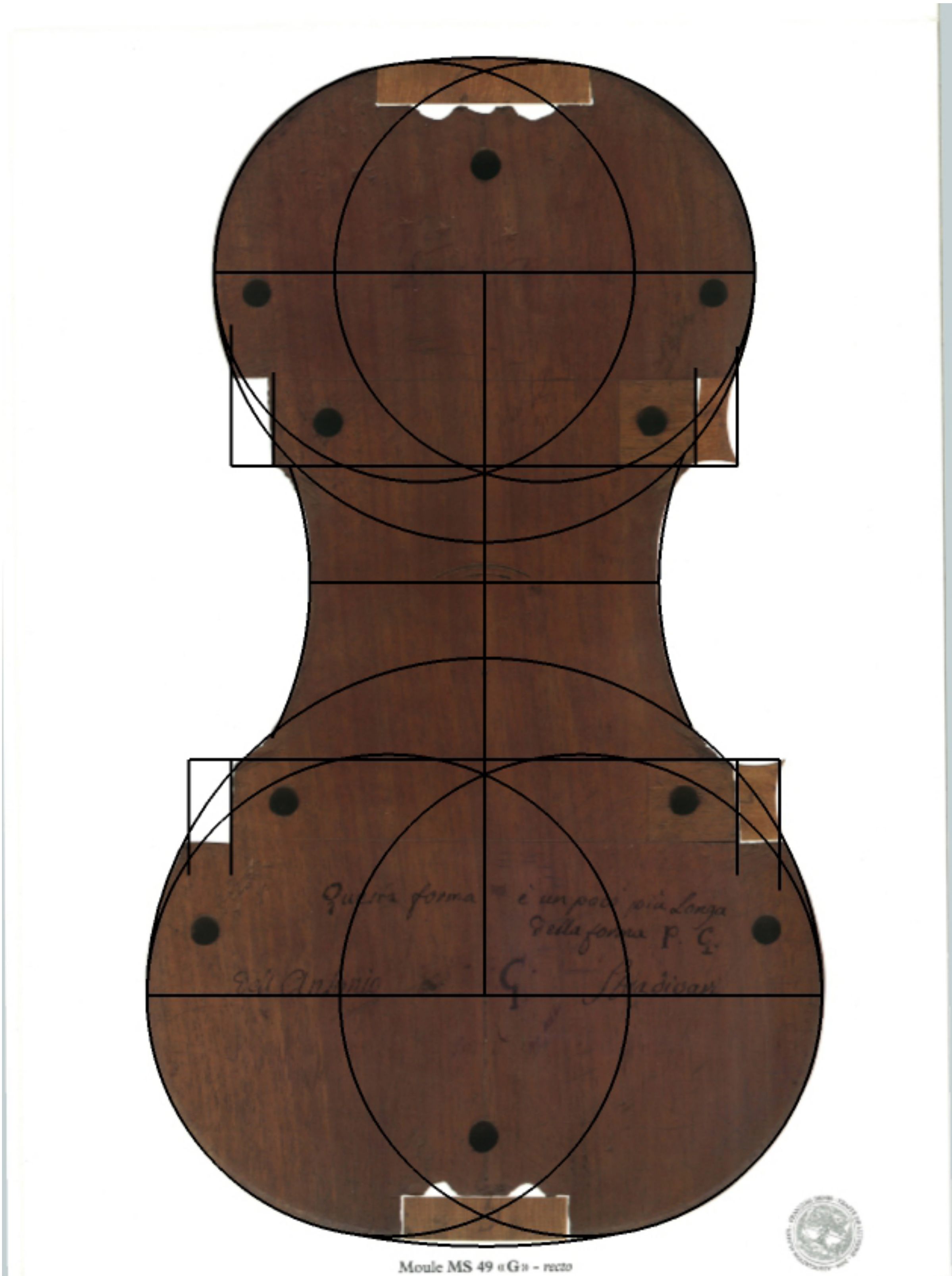

Figure 21. Strad G Form. (Photo used with permission from Francois Denis